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Garch-Midas Model used in Time Series Analysis

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Abstract
MIDAS regression in the variance forecasting problem has implications for option pricing and risk management. MIDAS stems from distributed lag models and makes it possible to estimate a model using variables measured at different frequencies. Forecasts are compared with realized volatility and accuracy is evaluated using a Quasi-likelihood loss function and Diebold Mariano test.

Introduction
Midas (Mixed Data Sampling) is a regression-based Garch model. MIDAS provides a framework for incorporating financial series and macroeconomic variables sampled with various frequencies. The GARCH-MIDAS model is a novel component GARCH model that incorporates macroeconomic data directly into the definition of the long-term component. The prior studies are largely restricted to variables like short-term interest rates, term premiums, and default premiums, for which daily data are accessible, as the analyses of the time-varying volatility are mostly dependent on high-frequency data. The effects of factors like inflation and the unemployment rate on volatility have not been thoroughly studied. The Garch-Midas model enhances the model's predictive power, especially for long term variance components. Furthermore, the initial principal component addition to the Garch-MIDAS model surpasses all other parameters. The short-term interest rate and the default rate outperform the other macroeconomic variables when they are included in the MIDAS equation. The equation is

$$r_{i,j} = \mu + \sqrt{\tau_i} g_{i,t} \in_{i,t} , \qquad \forall_i = 1,\ldots\ldots,N_t$$

$$\in_{i,t}/\emptyset_{i-1,t} \sim \mathrm{N}\,(0,1)$$

Where N_t is the number of trading days in month t and $\emptyset_{i-1,t}$ is the information set up to (i −1) th day of period t.

MIDAS (Mixed Data Sampling), a regression method that enables the integration of data from several frequencies into a single model. Macroeconomic data that is only seen at lower frequencies, such as monthly or quarterly, can be combined with high-frequency return data. Garchmidas's model is used to analyse market volatility. This approach separates the long-term and short term components of the conditional variance. Through the long term component, the low-frequency variables have an impact on the conditional variance.

Estimation Method of GARCH MIDAS MODEL
(a) **Various model specifications:** There are three different model specifications of the long-term variance component. The three specifications are:

(i) **The RV model:** Used the monthly realized volatility (RV) in the long-term component of the variance and no economic variables in the model.

(ii) **The RV** + X^l+ X^v **model:** Here, we augment the model by adding both the level and the variance of an economic variable to the MIDAS equation, τ_t. This modification is supposed to capture the information explained by both the macroeconomic factor and the monthly RV.

(iii) **The X^l+ X^v model:** In this specification, we only study the effect of macroeconomic variables, both level and variance, on the long-term variance component, i.e. equation for τ_t.

(b) GARCH(*p*, *q*) model specification : The lag length *p* of a GARCH(*p*, *q*) process is established in three steps:

(i) Estimate the best fitting AR(*q*) model:

$$Y_t. = a_0 + a_1 y_{t-1} + \ldots\ldots\ldots\ldots + a_q\, y_{t-q} = a_0 + \sum_{i=1}^{q} a_i\, y_{t-i} + \epsilon_t$$

(ii) Compute and plot the autocorrelations of ϵ^2 by

$$\rho = \frac{\sum_{t=i+1}^{T}({\epsilon_t}^2-{\sigma_t}^2)({\epsilon_{t-1}}^2-{\sigma_{t-1}}^2)}{\sum_{i=1}^{T}({\epsilon_t}^2-{\sigma_t}^2)}$$

(iii) The asymptotic, that is for large samples, standard deviation of $\rho(i)$ is $1/\sqrt{T}$. Individual values that are larger than this indicate GARCH errors. To estimate the total number of lags, uses the Ljung-Box test until the value of these are less than10% significant. The Ljung-Box Q-statistic follows $\chi^2$with n degrees of freedom if the squared residuals ϵ_t^2 are uncorrelated. It is recommended to consider up to T/4 values of n. The null hypothesis states that there are no ARCH or GARCH errors. Rejecting the null thus means that such errors exist in the conditional variance.

(c) Maximum likelihood estimation of GARCH-MIDAS model: The model parameters are estimated using the greatest likelihood method. The GARCH-MIDAS model's likelihood function contains a considerable number of parameters, and traditional optimization procedures do not necessarily lead to a global optimum. Use the simulated annealing method to estimate. Even for exceedingly complex issues, this method is very reliable and rarely fails. The optimum value of the likelihood function increases with the number of lags and converges to its highest level. Maximum values of the likelihood function are obtained by utilising various lags in the MIDAS equation.

Process of GARCH MIDAS Model: The kurtosis $K^{MG}$of an M-GARCH process is given by $K^{MG} = \frac{E[{\tau_t}^2]}{E[\tau_t]^2} . K^{GA} \leq K^{GA}\ Where, K^{GA} = K.E[{g_{i,t}}^2]$ is the kurtosis of the nested GARCH process and where the equality holds if and only if τ_t constant.. If $I_t = 1, the\ ACF, {\rho_k}^{MG}(\varepsilon^2)$ = corr$({\varepsilon_t}^2, \varepsilon_{t-k}^2)$ = ${\rho^\tau}_k \frac{Var(\tau_t)}{Var\,({\varepsilon_t}^2)} + {\rho^{GA}}_k \frac{Var(g_t Z_t^2)}{var\,({\varepsilon_t}^2)} ({\rho^\tau}_k\, var(\tau_t) + E[\tau_t]^2)$ with ρ_k^τ = corr$(\tau_{t,}\, \tau_{t-k})$ and ${\rho^{GA}}_k$ = Corr $(g_t Z_t^2, g_{t-k} Z_{t-k}^2) = (\alpha + \gamma/2 + \beta)^{k-1} K + \frac{(\propto + \gamma/2)(1-(\alpha+\gamma/2)\beta - \beta^2}{(1-(\alpha+\gamma/2)\beta-\beta^2)}$

If $I_t = 1, the\ ACF, {\rho_k}^{MG}(\sigma^2)$ of ${\sigma_t}^2$ is given by ${\rho_k}^{MG}(\sigma^2)$ = corr$({\sigma_t}^2, \sigma_{t-k}^2)$ = ${\rho^\tau}_k \frac{Var(\tau_t)}{Var\,({\sigma_t}^2)}$ + ${\rho^{GA}}_k \frac{Var(g_t Z_t^2)}{var\,({\sigma_t}^2)} ({\rho^\tau}_k\, var(\tau_t) + E[\tau_t]^2)$ with ρ_k^τ as before and p_k^g= corr$(g_{t,}\, g_{t-k})$ = $(\alpha + \gamma/2 + \beta)^k$.

If $\varepsilon_{k,t+1}^2$ follows a GARCH-MIDAS process, and $h_{k,t+1|t\ = It+1, g_{k,t+1}|}$ then the population ${R^2}_k$ of the MZ regression is given by

$R_k^2 = \frac{Var((h_{k,t+1,}|t)}{Var(\varepsilon_{k,\ t+1}^2)}$

If $\varepsilon_{k,t+1}^2$ follows a GARCH-MIDAS process, and $h_{1,t+1|t}=\tau_{t+1}g_{1,t+1}$, then the population R_1^2 of the MZ regression is given by

$$R_1^2 = \frac{[1-(\alpha+\gamma/2+\beta)^2 E[\tau_{t+1}^2]-[1-(\alpha+\gamma/2+\beta)^2 K-2(\alpha+\gamma/2)\beta-\beta^2)]E[\tau_{t+1}]^2]}{[1-(\alpha+\gamma/2+\beta)^2 E[\tau_{t+1}^2]K-[1-(\alpha+\gamma/2+\beta)^2 K-2(\alpha+\gamma/2)\beta-\beta^2)]E[\tau_{t+1}]^2]}$$

Estimation strategy of GARCH-MIDAS Model: Estimations are based on the daily observations of returns, while monthly frequency is used in the MIDAS equation to capture the long-term component. The "realised volatility" is the preferred measure of the monthly variance, but since daily data are not available for most macroeconomic variables, it is not possible to use this measure. Choose the squared initial differences to represent the economic variables' variance.

Mean Square Error (MSE) and the Mean Absolute Error (MAE): There are several ways to evaluate a model's ability to forecast variance, including comparing expected variance to realised monthly volatility, which is calculated as the sum of the squared daily log returns over each month. Used two loss functions, the Mean Square Error (MSE) and the Mean Absolute Error (MAE), defined as

$$\text{MSE} = \frac{1}{T}\sum_{i=1}^{T} \ ({\sigma^2}_{t+1} - E_t\,({\sigma^2}_{t+1}))^2$$

$$\text{MAE} = \frac{1}{T}\sum_{i=1}^{T} \left|({\sigma^2}_{t+1} - E_t({\sigma^2}_{t+1}))^2\right|$$

MSE is a quadratic loss function and weights large prediction mistakes more heavily than the MAE measure; it is appropriate when large errors are more harmful than small errors.

Diebold and Mariano test: DM test, to compare the prediction accuracy of two competing models,

$$\text{DM} = \frac{E(d_t)}{\sqrt{Var(d_t)}} \sim \text{N}(0,1)$$

$$d_t = {e^2}_{A,t} - {e^2}_{B,t}$$

Where $e_{A,t}$and $e_{B,t}$are prediction error of two rival models *A* and *B*, respectively, and E(*dt*) and Var(*dt*) are mean and the variance of the time-series of *dt*, respectively.

In addition to these measures we run the following regression of the realized variance on the predicted variance

$${\sigma_t}^2 = \text{a} + \text{b}\ E_t({\sigma^2}_{t+1}) + u_t$$

If the predicted variance has some information about the future realised volatility, then the parameter *b* should be significantly different from zero. For an unbiased prediction, expect parameter *a* to be zero and parameter *b* to be equal to one.

Assumptions of GARCH MIDAS Model: Let $Z_{i,t}$ be indentically independently distributed with E[$Z_{i,t}$] = 0 , E[$Z_{i,t}^2$] = 1 and 1< K< ∞ , where K = E[$Z_{i,t}^4$]**.** Assume that $\propto\ > 0$, $\propto\ + \gamma > 0$, $\beta \geq 0$ and $\propto\ + \gamma/2 + \beta\ < 1$.Moreover, the parameters satisfy the condition $(\alpha + \gamma/2)^2\ K + 2(\alpha + \gamma/2)\,\beta + \beta^2 < 1$. Let f(.) > 0 be a measurable function and X_t be a strictly stationary and ergodic time series with E[$|X_t|^q$, Where q is sufficiently large to ensure that E[${\tau_t}^2$] < ∞ . X_t is independent of Zi,t−j for all t, i and j.

Advantage of the GARCH-MIDAS model: The advantage of the Garch-MIDAS model is that it enables us to directly analyse how macroeconomic variables affect stock volatility by coupling daily observations of stock returns with macroeconomic variables collected at lower frequencies. to investigate how macroeconomic factors affect the volatility of the stock market. to determine whether including economic variables can enhance the classic volatility models' capacity for predicting. Garch-MIDAS is used to separate return volatility into short-term and long-term components, the latter of which is influenced by smoothed realised volatility. to cut back on the number of parameters while improving computation speed.

Forecasting of GARCH –MIDAS model: All other parameters are outperformed by the Garch-MIDAS model that incorporates the first principal component. Its close relationship to the variables short-term interest rate and default rate, which makes the first principal component an excellent proxy for the economic cycle, may be the cause of its high performance. The forecasting ability of the GARCH-MIDAS model is compared with a simple GARCH (1.1) model,

$$r_t = \mu + n_t \ , n_t = \sigma_t z_t \ , z_t \ \sim \mathrm{N}(0,1)$$
$$\sigma_t{}^2 \ = \omega + \propto n_t{}^2 + \ \beta \ \sigma_{t-1}^2$$

Using monthly observations to forecast long-term volatility and daily observations to forecast short-term volatility. Compare the monthly realised volatility, calculated as the sum of daily squared returns in a given month, to the out-of-sample monthly variance forecasts of the Garch-MIDAS and Garch models. Comparing the anticipated daily total variance of the GARCH-MIDAS and GARCH models with the actual daily volatility, expressed as squared returns, can help determine how well the models can predict the near term.

Summary and Conclusion

MIDAS regression makes use of information contained in higher-frequency data as it is significantly better at forecasting for all examined data and frequencies. Results from VaR estimation are not as clear. MIDAS significantly overestimates tail size, even with the assumption of normally distributed conditional returns. It is calculated as the sum of daily squared returns in month t plus out-of-sample forecasts of monthly variances from the Garch-Midas and Garch models.

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